8. D. B. Spalding, "A novel finite-difference formulation for differential expressions involving both first and second derivatives," Int. J. Num. Methods Eng., No. 4, 551-559 (1972).
9. G. N. Abramovich, Theory of Turbulent Jets [in Russian], Nauka, Moscow (1960).
10. G. N. Abramovich, Applied Gas Dynamics [in Russian], Nauka, Moscow (1969).
11. Kim, Klein, and Johnston, "Study of the reattachment of a turbulent shear layer: flow about an inverse recess," Trans. ASME Ser. D, 102, No. 3, 124-132 (1980).

STABILITY LIMIT OF THERMALLY DRIVEN OSCILLATIONS IN A TUBE OF VARIABLE CROSS SECTION

V. A. Sysoev and S. P. Gorbachev

UDC 621.59:534.1:546.291

The stability limit for helium in a tube of variable cross section is established and experimentally confirmed.

As is known, in a nonisothermal tube with a closed heated end and the other end placed in a cryostat with liquid helium, thermally driven oscillations may occur. Here, the heat flow to the liquid helium may increase by an order or more [1, 2].

One method of studying this phenomenon is determining the range of parameters of the system within which these oscillations may take place, i.e. finding the necessary condition for occurrence of the oscillations or solutions of the stability problem. This problem was examined in [3-6] for tubes of constant cross section. The present work studies the stability of oscillations in tubes the radius of which changes along their length and, in particular, increases intermittently. This corresponds to a tube composed of tubes of different diameter or to the attachment of closed volumes to the ends of a tube. The model and methods developed in [3, 4] will be used to construct the mathematical model and solve the problem.

These works made assumptions regarding triviality and did not consider: 1) the radial gradient of the acoustic pressure; 2) the radial change in the mean temperature and viscosity; 3) heat flow and friction due to axial gradients. For a tube of variable cross section, we add the assumptions that the section of an elemental streamtube changes in proportion to the section of the tube, i.e. $r(x)\Delta r(x) \sim r_0^2(x)$, and that the radial component of the velocity of the gas can be ignored. The latter assumption is valid for tubes with a small change in section along their length or when the absolute value of the velocity is low and, thus, its radial component is small (for example, in the case of a sudden contraction or expansion of the tube section near the closed end). Allowing for these assumptions, the linearized system of equations for the tube of variable cross section has the form

$$\frac{\partial \rho}{\partial \tau} + \frac{\rho_0}{r_0^2(x)} \frac{\partial}{\partial x} (r_0^2(x) U) + \frac{\partial \rho_0}{\partial x} U = 0, \tag{1}$$

$$\frac{\partial U}{\partial \tau} + \frac{1}{\rho_0} \frac{\partial P}{\partial x} = \nu \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right), \tag{2}$$

$$\frac{\partial T}{\partial \tau} + U \frac{dT_0}{dx} - \frac{1}{\rho_0 c_p} \frac{\partial P}{\partial \tau} = \frac{\lambda}{\rho_0 c_p} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right). \tag{3}$$

The system is closed by the linearized equation of state

$$T = \frac{P}{R\rho_0} - \frac{T_0}{\rho_0} \rho. \tag{4}$$

Balashikhinsk Scientific-Engineering Association of Cryogenic Machine Construction. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 1, pp. 31-35, January, 1984. Original article submitted July 30, 1982.

0022-0841/84/4601-0023$08.50

Since system (1)-(4) permits a solution of the form (P, ρ, T, U) = (F_1, F_2, F_3, F_4) exp (iωτ), then we can replace ∂/∂τ by iω. After transformation, this system can be reduced to a single equation in pressure

$$r_0^2(x)[1+(\gamma-1)f_1^*]P+\frac{d}{dx}\left[\frac{a^2}{\omega^2}r_0^2(x)(1-f_1)\frac{dP}{dx}\right]-r_0^2(x)\frac{a^2}{\omega^2}\frac{f_1^*-f_1}{1-\sigma}\frac{1}{T_0}\frac{dT_0}{dx}\frac{dP}{dx}=0. \quad (5)$$

In Eq. (5)

$$f_1=-\frac{i}{\eta}\frac{J_0'(i\eta)}{J_0(i\eta)};\quad f_1^*=f_1(i\eta\sqrt{\sigma});\quad \eta=\left(\frac{i\omega}{\nu}\right)^{1/2}r_0(x),$$

where J_0 is a zeroth-order Bessel function. We rewrite Eq. (5):

$$g_1P+\frac{d}{dx}\left(g_2\frac{dP}{dx}\right)-g_3\frac{dP}{dx}=0,$$

and, having transformed it to the form

$$g_1EP+\frac{d}{dx}\left(g_2E\frac{dP}{dx}\right)=0, \quad (6)$$

where

$$E=\exp\left(\int\frac{g_3}{g_2}dx\right),$$

we represent it in the form of the system

$$P=-\frac{1}{g_1E}\frac{d\psi}{dx};\quad \psi=\frac{g_1E}{k^2}\frac{dP}{dx};\quad k^2=\frac{g_1}{g_2}. \quad (7)$$

It was shown in [3] that the singularity and discontinuity of the functions P and ψ are eliminated from system (7). Thus, the following relations are satisfied for the points of discontinuity of the function at the section x = l' and the point of discontinuity of the temperature at x = l:

$$P(x-0)=P(x+0);\quad \psi(x-0)=\psi(x+0). \quad (8)$$

We will use these relations to solve the problem of the stability of the oscillations in a tube with a sudden change in its section at the ends (Fig. 1). To simplify the problem we will assume that the nonisothermality of the tube is given by a piecewise-constant temperature profile. The oscillations in such a tube are governed by the following mechanism. With a shift in the position of thermal equilibrium from the wall toward the warm end, the gas is simultaneously compressed and heated and its pressure increases. The gas expands and moves toward the cold end, were heat being transferred from the gas to the wall of the tube. The pressure in the tube becomes lower than in the cold volume, and a reverse flow of gas to the warm end is set up, i.e. the cycle is repeated.

The boundary conditions are assigned as follows:

$$\left.\frac{dP}{dx}\right|_{x=-l_0}=0;\quad \left.\frac{dP}{dx}\right|_{x=L}=0. \quad (9)$$

The general solution of Eq. (5) for a tube segment with a constant cross section and constant temperature is written thusly:

$$P=A\sin kx+B\cos kx,$$

where

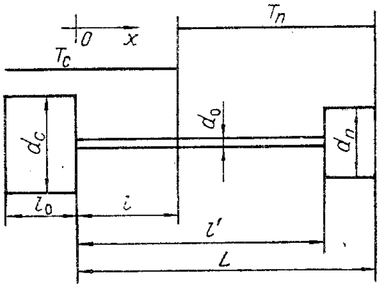

Fig. 1

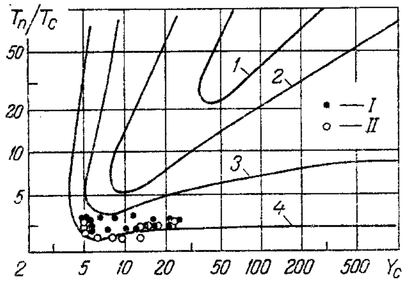

Fig. 2

Fig. 1. Diagram of nonisothermal pipe with containers at its ends.

Fig. 2. Stability limit for helium with different ratios of the cross sections of the volume and the tube M: 1) M = 0.5; 2) 1; 3) 2; 4) 10; $l' = l$, $(L-l)/l = 0.5$. Experiment: d_n = 36 mm; d_0 = 3.6 mm; l = 0.5 m; $l' = l$; $L-l$ = 0.25 m; I) oscillations present; II) oscillations absent.

$$k = \frac{\omega}{a}\left[\frac{1+(\gamma-1)f_1^*}{1-f_1}\right]^{1/2}.$$

Writing this solution for each segment with a constant temperature and section and satisfying boundary conditions (9) and compatibility conditions (8) at the points $x = l$ and $x = l'$, we obtain a system of six linear homogeneous equations which has a nontrivial solution if its determinant is equal to zero. This condition gives us the following equation in the complex variable η:

$$G_{c2}\frac{\operatorname{ctg} k_{c2}l}{k_{c2}}\left[\frac{1+q_c \operatorname{tg} k_{c2}l}{1-q_c \operatorname{ctg} k_{c2}l}\right] = G_{n1}\frac{\operatorname{tg} k_{n1}(l'-l)}{k_{n1}}\left[\frac{1+q_n \operatorname{ctg} k_{n1}(l'-l)}{1-q_n \operatorname{tg} k_{n1}(l'-l)}\right], \tag{10}$$

where

$$q_c = \frac{F_{c2}}{F_{c1}}\frac{k_{c1}}{k_{c2}}\operatorname{ctg} k_{c1}l_0; \quad q_n = \frac{F_{n2}}{F_{n1}}\frac{k_{n1}}{k_{n2}}\operatorname{tg} k_{n2}(L-l');$$

$$F = g_1; \quad G = g_1 E.$$

Equation (10) transforms into a system of two equations in the dimensionless parameters $Y_c = \sqrt{\omega/\nu_c}\,r_c$ and $\lambda_c = \omega l/a_c$. This system has a set of solutions, but the stability limit corresponds to the minimum value of λ_c. Numerical results were obtained for the case when the volume on the cold end is infinitely large — which is the case for the tube placed in the cryostat.

The stability limit (Fig. 2) has two branches delimiting the region of unstable oscillations on the left and right and is determined by the value of the parameter Y_c, which is connected with the geometric parameters of the tube by the relation $Y_c = \Phi\lambda_c^{1/2}$ and with the pressure inside the tube through the kinematic viscosity ν_c. Here, the right branch of the stability limit is associated with excitation of the oscillations as the temperature of the warm end increases, while the left branch is associated with damping of the oscillations. For M = 1, the stability limit corresponds to the calculations in [4] and the experimental data in [7]. It follows from Fig. 2 that the expansion of the warm region of the tube significantly lowers the ratio of the temperatures of the warm and cold ends at which the oscillations are excited.

On the other hand, it follows from Fig. 3 that an increase in the ratio of the sections of the warm and cold segments M causes a decrease in the frequency parameter λ_c and, thus, the parameter Y_c. Here, the latter moves in the direction of the left branch of the stability limit. The crossing of the left branch is associated with damping of the oscillations.

Thus, the effect of expansion of the tube at the warm end is of a dual nature. On the one hand, the expansion helps excite the oscillations. On the other hand, it may damp them.

This duality is due to the fact that expansion of the warm part of the pipe is accompanied by a decrease in friction, and the increase in the heat exchange surface and decrease

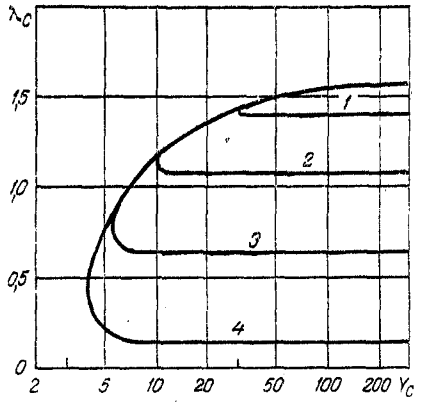

Fig. 3

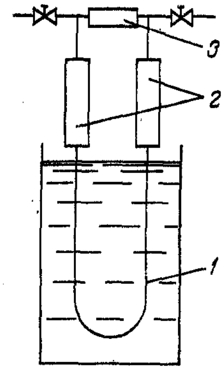

Fig. 4

Fig. 3. Frequency parameter λ_c with different values of M: 1) M = 0.5; 2) 1; 3) 2; 4) 10.

Fig. 4. Diagram of the stand.

in oscillation frequency improve heat exchange between the gas and wall. However, when heat exchange and friction reach limiting values and become steady, the increase in volume begins to damp the oscillations. At the same time, contraction of the warm part of the tube raises the lower branch of the stability limit and lowers the upper branch, which reduces the size of the region of instability, i.e. facilitates damping of the oscillations.

The results of the calculations were checked experimentally. The testing element (Fig. 4) was a symmetrical U-shaped tube 1 with volumes 2 on its ends. With antisymmetrical oscillations, these components correspond to the diagram presented in Fig. 1 if the volume on the cold end is infinitely large, since the boundary condition P(0) = 0 is satisfied in both cases. The antisymmetrical oscillations were recorded by an induction-type pressure transducer 3, which recorded the oscillations between the volumes. The tube was lowered into liquid nitrogen, while the temperature of the volumes was established with in the range 230-250°K with the aid of insulation and resistance thermometer bridges. The results of the theory and experiment, shown in Fig. 2, are in satisfactory agreement.

Thus, extension of the model in [3] to a tube of variable cross section allowed us to show that the effect of the volume on the warm end on the stability of the thermally driven oscillations is of a dual nature. On the one hand, the volume helps excite the oscillations. On the other hand, it may damp them. We theoretically showed and experimentally proved the possibility and exciting oscillations of helium in a tube with a temperature of about 80°K on the cold end and about 250°K at the warm end. It was concluded earlier [4, 7] that the minimum ratio of the temperatures of the warm and cold ends at which oscillations are excited is 5.5.

NOTATION

x, r, axial and radial coordinates; τ, time; ρ_0, P_0, T_0, mean values of density, pressure, and temperature; ρ, P, T, U, nonsteady components of density, pressure, temperature, and velocity; λ, thermal conductivity of the gas; c_p isobaric heat capacity; $\gamma = c_p/c_v$; σ = Prandtl number; ν, kinematic viscosity of the gas; a, speed of sound; $r_0(x)$, radius of the tube at the section x; M, ratio of the radii of the warm and cold ends; ω, frequency of the oscillations, 1/sec; $\Phi = (a_c/\nu_c)^{1/2} d_0/i^{1/2}$, dimensionless parameter. Indices: c1, c2, values of the parameters at the temperature T_c and the sections d_c, d_0; n1, n2, at the temperature T_n and sections d_0, d_n.

LITERATURE CITED

1. A. G. Shifel'bain, "Determination of heat flow to a helium conduit with a nitrogen shield," Dep. No. 555, TsINTIKhIMneftemash, No. 11 (97), 87 (1979).
2. J. D. Bannister, "Spontaneous pressure oscillations in tubes connecting liquid helium reservoirs to 300°K environments," in: Liquid Helium Technology (1966), pp. 127-135.
3. N. Rott, "Damped and thermally driven acoustic oscillations in wide and narrow tubes," ZAMP, 20, No. 2, 230-243 (1969).

4. N. Rott, "Thermally driven acoustic oscillations. Pt. II. Stability limit for helium," ZAMP, 24, No. 1, 54-72 (1973).
5. S. I. Sergeev, O. M. Popov, and I. P. Vishnev, "Thermally driven oscillations of helium in unevenly heated tubes," Inzh.-Fiz. Zh., 36, No. 2, 344-352 (1979).
6. S. I. Sergeev, O. M. Popov, and I. P. Vishnev, "Thermally driven oscillations of helium in cryogenic apparatus," Inzh.-Fiz. Zh., 32, No. 4, 691-695 (1977).
7. T. Yazaki, A. Tominaga, and Y. Narahara, "Stability limit for thermally driven acoustic oscillation," Cryogenics, 19, No. 7, 393-396 (1979).

RELAXATION EFFECTS IN THE PROPAGATION OF SHOCK WAVES IN ANOMALOUS OILS

K. V. Mukuk

UDC 532.135:622.323

Experimental results are presented from a study of the propagation of wave pertubations in anomalous oils exhibiting relaxation phenomena.

Nonequilibrium phenomena occurring in anomalous oils [1, 2] containing a substantial quantity of paraffinous and resinous fractions necessitate the use of rheologically complex models to describe their flow [3]. The viscoelastic and thixotropic properties of the oil have a significant effect on wave processes and make it necessary to allow for them in examining transient conditions in hydraulic lines.

The structure and dynamics of propagation of pressure perturbations in anomalous oils at different temperatures was studied on a shock tube of a design similar to that used in [4]. The tube was developed by the Institute of Thermophysics of the Siberian Branch of the Soviet Academy of Sciences to study wave processes in gas—liquid systems. The shock tube consists of a high-pressure chamber (HPC) and was connected in the experiment to a diaphragm block. The pressure in the HPC was created by means of a special cylinder and was recorded by two regulators. The working part of the tube, 1654 mm long and 33 mm in diameter, was equipped with three LKh 601 piezoceramic pressure transducers with a resonance frequency no lower than 50 kHz. The lower limit of the frequency-independent characteristic was 20 Hz. Shock waves with an intensity up to 1.5 MPa were created by the rupture of membranes made of aluminum foil. The structure and profile of the pressure change during propagation in the oil were recorded by two C8-3 recording oscillographs. The operation of the oscillographs was synchronized with the aid of a special triggering sensor. The working part of the tube was completely covered by a silicone sleeve and connected to a cryostat, ensuring the prescribed temperature conditions in the range from 0 to 60°C.

The tests were conducted on resinous and paraffinous oils at temperatures of 10-40°C. Special rheological investigations conducted under kinetic and dynamic conditions established that these oils exhibit anomalous viscosity characteristics and relaxation effects

TABLE 1. Rheological Characteristics of the Oils

t, °C	Paraffinous oil			Resinous oil		
	τ_0, Pa	η, Pa·sec	relaxation time, msec	τ_0, Pa	η, Pa·sec	relaxation time, msec
10	30	0,109	240	117	4,26	870
15	26	0,063	5	80	2,91	40
20	17	0,041	3	40	2,10	21
30	0	0,027	1	0	0,083	0,4
40	0	0,015	—	0	0,35	—

Middle-Asian Scientific Research and Design Institute of the Petroleum Industry. Ministry of the Petroleum Industry of the USSR, Tashkent. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 1, pp. 35-38, January, 1984. Original article submitted September 22, 1982.

0022-0841/84/4601-0027$08.50